A problem on complex root of quadratic equation

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Question If $a + 2i$ is a root of the quadratic equation $2x^2 + 6x + b = 0$, find the values of the real numbers a and b.

Method 1

$$2x^2 + 6x + b = 0 \qquad \ldots. \ (1)$$

By quadratic equation formula,

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(b)}}{2(2)} = \frac{-6 \pm \sqrt{36 - 8b}}{4} = -\frac{3}{2} \pm \frac{\sqrt{36 - 8b}}{4}$$

Hence $a + 2i = -\frac{3}{2} + \frac{\sqrt{8b - 36}}{4} i$ (note that one of the roots is rejected)

Compare real and imaginary parts,

$$\begin{cases} a = -\frac{3}{2} & \ldots.(2) \\ 2 = \frac{\sqrt{8b - 36}}{4} & \ldots.(3) \end{cases}$$

From (3),

$$8 = \sqrt{8b - 36} \qquad \Rightarrow 64 = 8b - 36 \qquad \Rightarrow b = \frac{100}{8} = \frac{25}{2}$$

$$\therefore \quad (a, b) = \left(-\frac{3}{2}, \ \frac{25}{2}\right)$$

Method 2

Since $a + 2i$ is a root of the quadratic equation

$$2x^2 + 6x + b = 0 \qquad \ldots. \ (1)$$

$$\therefore \quad 2(a + 2i)^2 + 6(a + 2i) + b = 0$$

$$2(a^2 + 4ai - 4) + 6a + 12i + b = 0$$

$$(2a^2 - 8 + 6a + b) + (8a + 12)i = 0$$

$$\therefore \quad \begin{cases} 2a^2 - 8 + 6a + b = 0 & \ldots.(2) \\ 8a + 12 = 0 & \ldots.(3) \end{cases}$$

From (3), $a = -\frac{3}{2}$ .... (4)

(4)↓(3), $2\left(-\frac{3}{2}\right)^2 - 8 + 6\left(-\frac{3}{2}\right) + b = 0 \quad \Rightarrow \frac{9}{2} - 8 - 9 + b = 0$

$\therefore \quad b = \frac{25}{2}$

Method 3

Since $a + 2i$ is a root of the quadratic equation

$2x^2 + 6x + b = 0$ …. (1)

$a - 2i$ is a also root of the quadratic equation .

(Complex roots occur in pair.)

The quadratic equation must be of the form :

$[x - (a + 2i)][x - (a - 2i)] = 0$

$[(x - a) + 2i][(x - a) - 2i] = 0$

$(x - a)^2 - (2i)^2 = 0$

$x^2 - 2ax + a^2 + 4 = 0$

$2x^2 - 4ax + 2a^2 + 8 = 0$ …. (2)

(1) and (2) are identical quadratic equation, and by comparing coefficients, we get:

$$\begin{cases} -4a = 6 & \text{....(3)} \\ 2a^2 + 8 = b & \text{....(4)} \end{cases}$$

From (3), $a = -\frac{3}{2}$ …. (5)

(5)↓(4), $2\left(-\frac{3}{2}\right)^2 + 8 = b \quad \Rightarrow \frac{9}{2} + 8 = b \quad \Rightarrow b = \frac{25}{2}$

Method 4

Since $a + 2i$ is a root of the quadratic equation

$2x^2 + 6x + b = 0$ …. (1)

$x = a + 2i$

$x - a = 2i$

Squaring, $(x - a)^2 = (2i)^2$ (note that squaring creates root)

$x^2 - 2ax + a^2 = -4$

$x^2 - 2ax + a^2 + 4 = 0$

$2x^2 - 4ax + 2a^2 + 8 = 0$ …. (2)

The subsequent part follows the same way as **Method 3** .

Method 5

Since $a + 2i$ is a root of the quadratic equation

$$2x^2 + 6x + b = 0 \qquad \ldots. \quad (1)$$

$a - 2i$ is a also root of the quadratic equation .

(Complex roots occur in pair.)

By Vieta theorem,

Sum of roots = $(a + 2i) + (a - 2i) = -\frac{6}{2}$ .... (2)

Product of roots = $(a + 2i)(a - 2i) = \frac{b}{2}$ .... (3)

From (2), $2a = -\frac{6}{2} \quad \Rightarrow a = -\frac{3}{2}$ .... (4)

From (3), $a^2 - (2i)^2 = \frac{b}{2}$

$\therefore \quad b = 2(a^2 + 4) = 2\left[\left(-\frac{3}{2}\right)^2 + 4\right] = \frac{25}{2}$.